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Insights from orthogonality for mathematics

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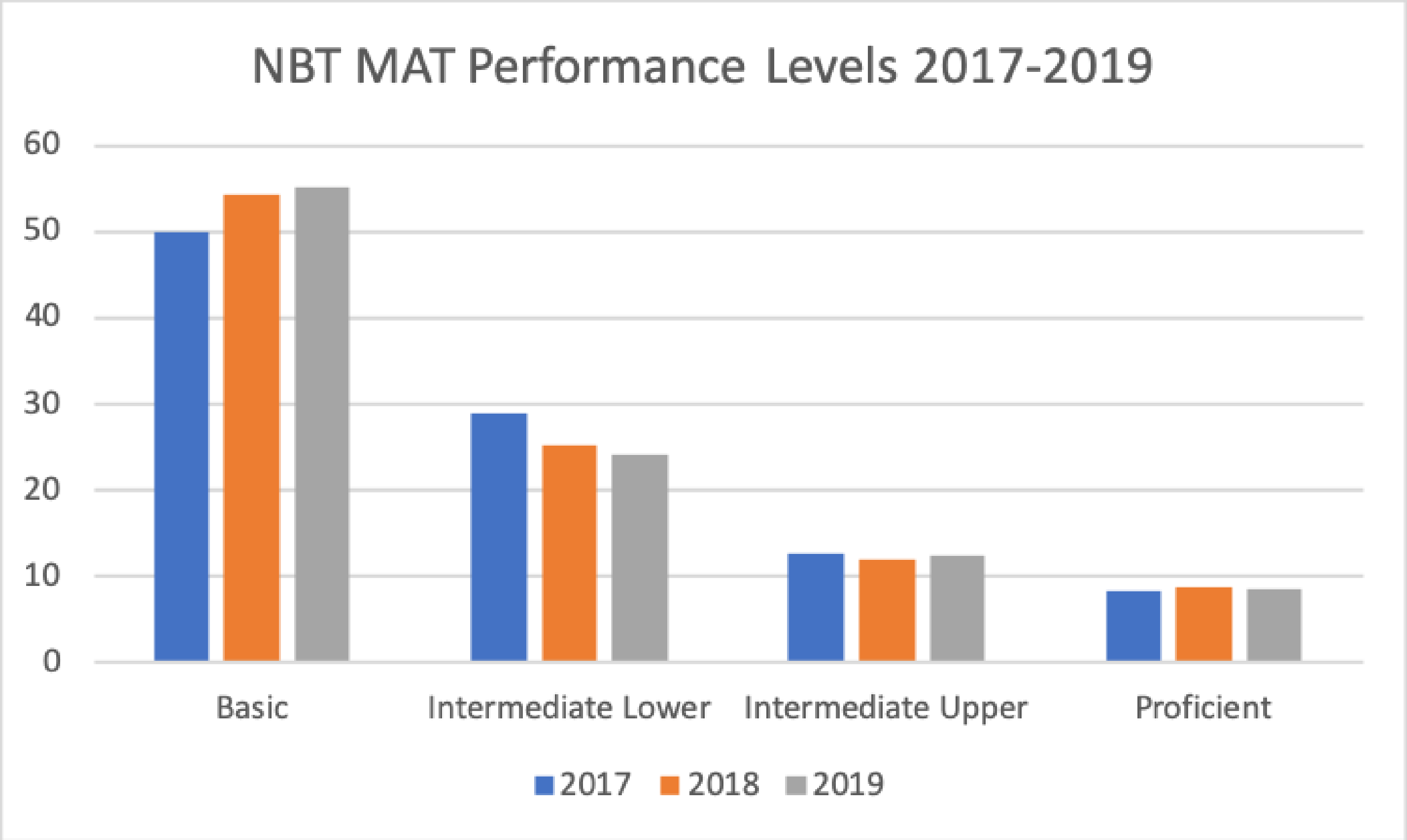
Rationale and Context

As many as ninety percent of the prospective students for tertiary education institutions in South Africa have no more than an intermediate level of mathematical proficiency.



CETAP Centre for Educational Testing for Access and Placement

NBT National Benchmark Tests Project

Rationale and Context

discontinuity between the outcomes of schooling and the demands of higher education, commonly known as the 'articulation gap'.

In the South African context the **articulation gap** between school and university is understood to involve depth of understanding, depth of learning, and level of competency with the acquired skills.

Council for Higher Education 2013; Scott et al 2007

Rationale and Context

discontinuity between the outcomes of schooling and the demands of higher education, commonly known as the 'articulation gap'.

In the South African context the **articulation gap** between school and university is understood to involve depth of understanding, depth of learning, and level of competency with the acquired skills.

Seeing this problem as an articulation gap (DoE 1997: 2.34), rather than just as student underpreparedness, opens up possibilities for positive action within higher education, because **a gap can be closed from either side.**

Council for Higher Education 2013; Scott et al 2007

Rationale and Context

How may the gaps in mathematical proficiency be addressed?

"The real test of how much and how well you know comes when you enter university. Here the **rules are different**. It will not help you in a good university to memorise and repeat facts. What will be tested is your ***ability to think critically, independently and thoughtfully***. The smart scholars among you will, for the first time, experience difficulty in one or more university subjects."

Jonathan Jansen's article "*Dear Grade 12 pupil: Let's talk about your matric pass*", 2017

Rationale and Context

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Orthogonality is key!

Research Question

What is the essence of Mathematical proficiency?

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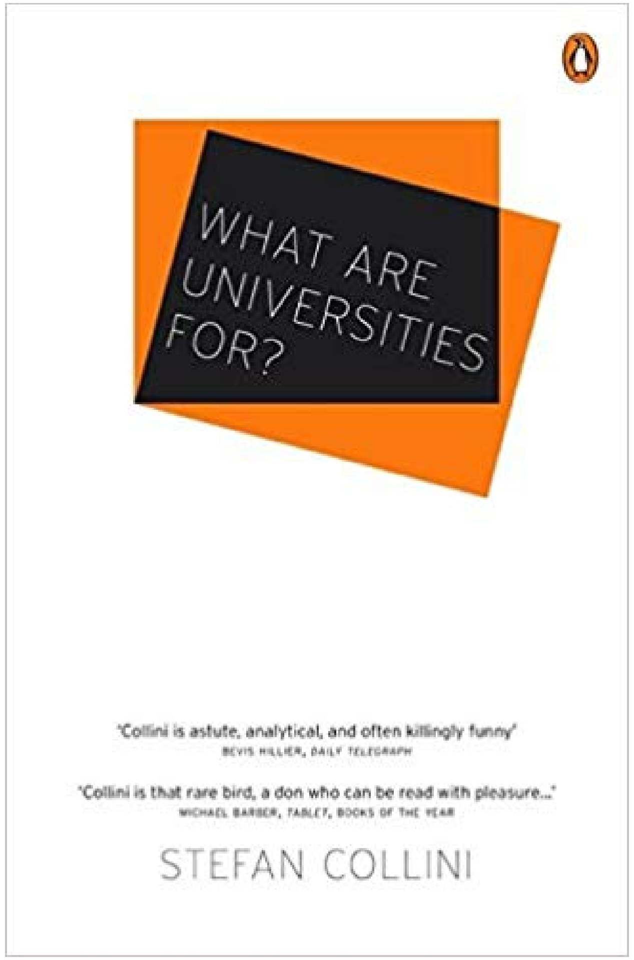
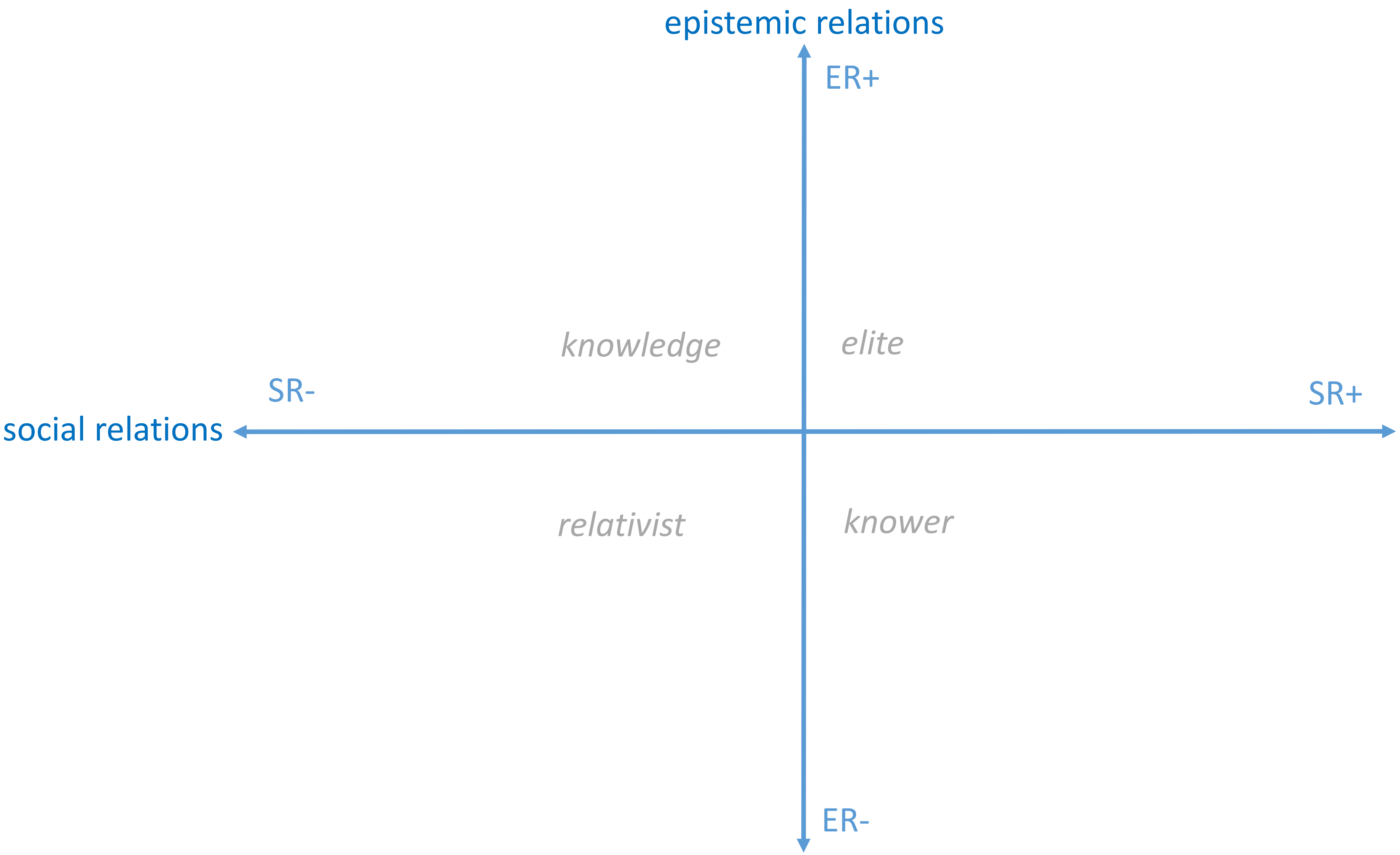
Perhaps several binary oppositions are at play:

- pure vs applied
- mental vs physical
- verbal vs visual
- algebraic vs geometric
- human activity vs mathematical idea
- intuition vs formal mathematical representation
- disciplinary vs cross-disciplinary
- specialisation vs generalization
- **knowledge vs skills**



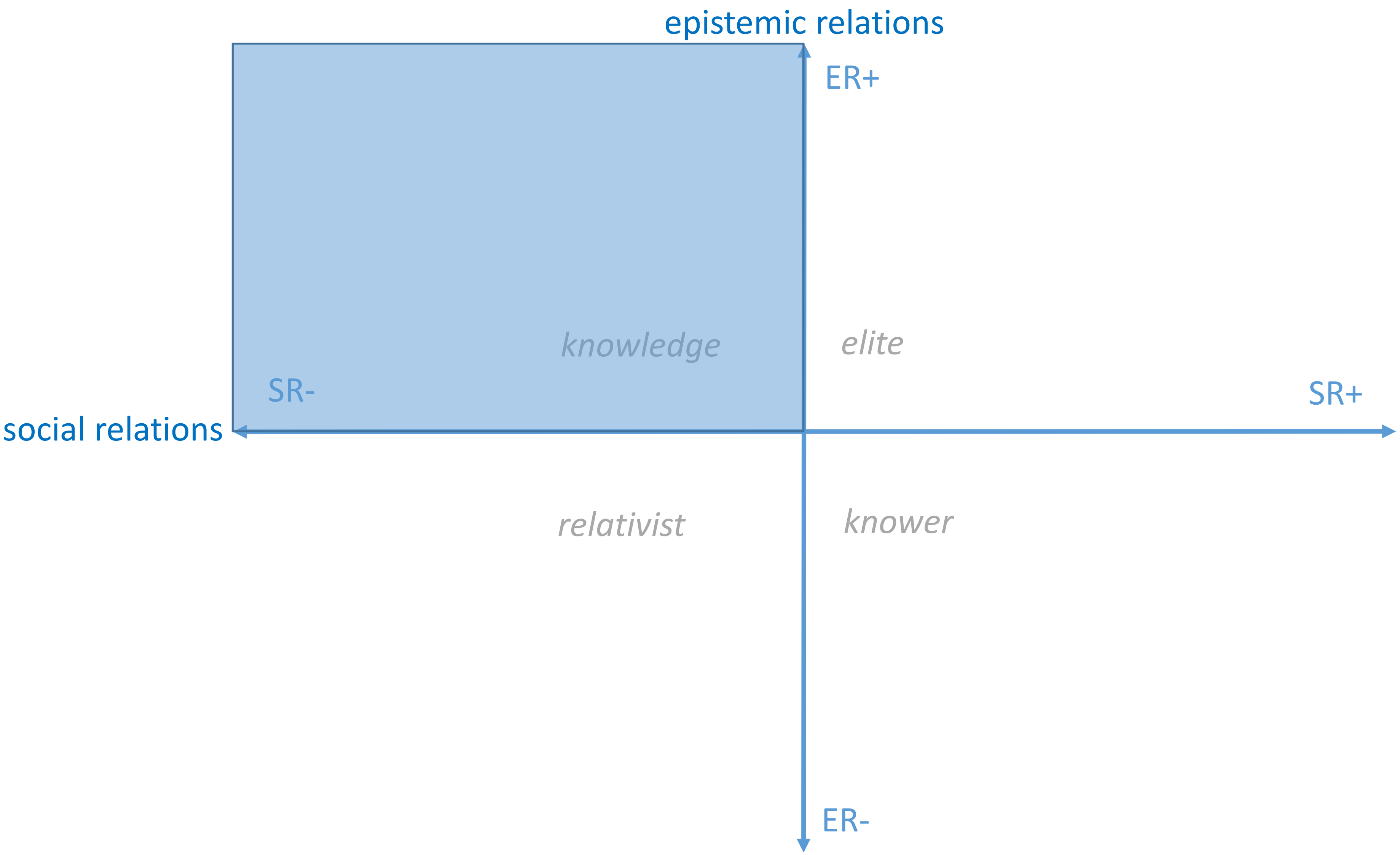
What are Universities For? (Stefan Collini, 2012)

A conceptual framework



Specialization dimension of Legitimation Code Theory Maton 2014)

A conceptual framework



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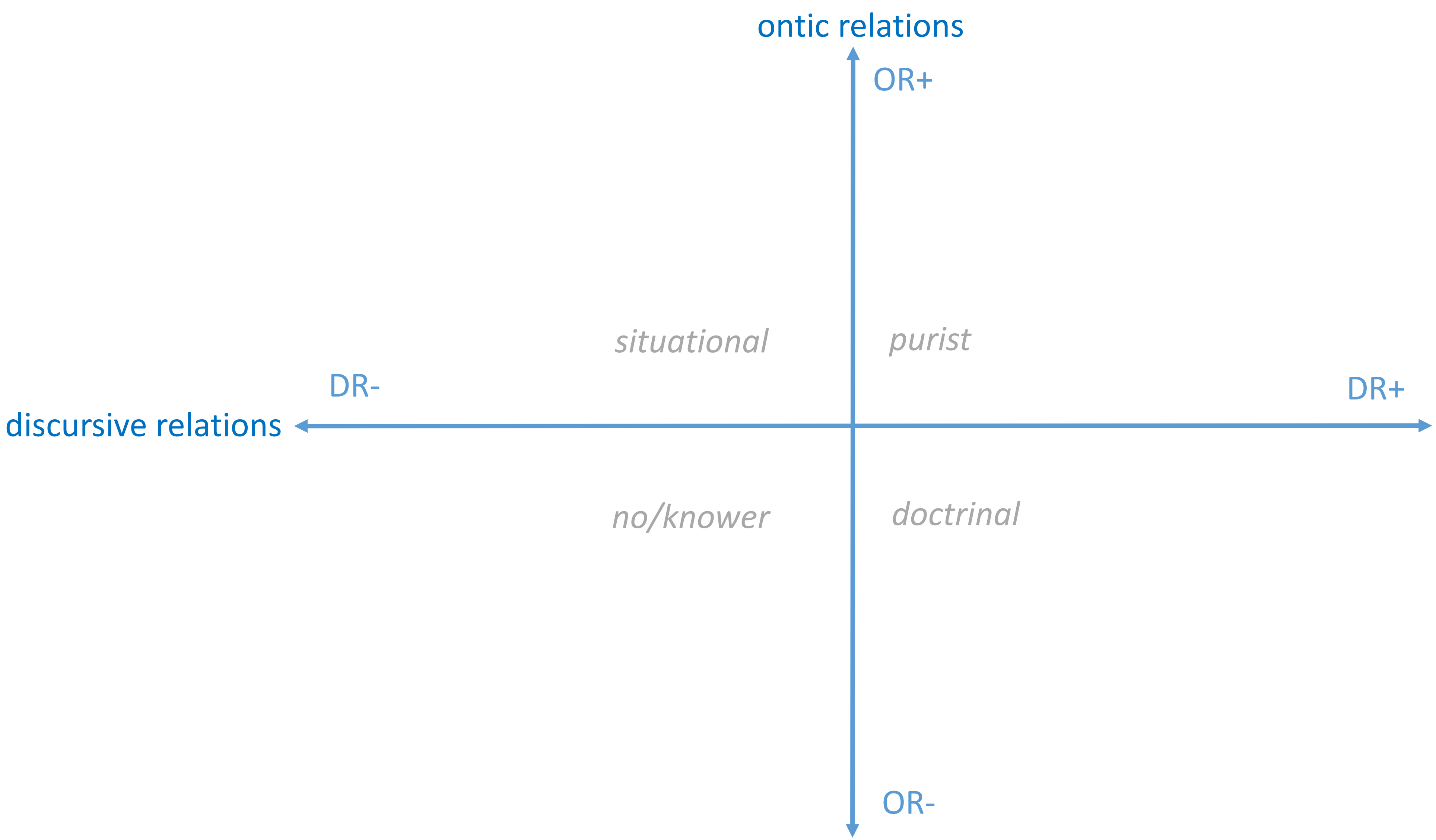

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1918 · 2018

Reflective thinking turns experience into insight.

How Successful People Think: Change your thinking, change your life (John C Maxwell, 2009).

Insights: Knots

DR -

DR+

discursive relations ⟷ Know-how

Insights: Knots

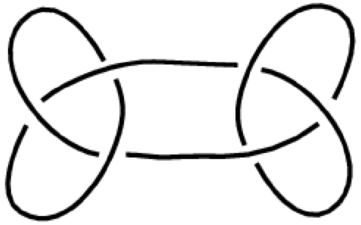
DR $^{-}$ DR $^{+}$

discursive relations ⟵⟶ Know-how

Insights: Knots

DR $^{-}$ DR $^{+}$

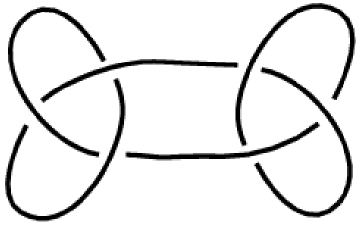
discursive relations ↔ Know-how



Insights: Knots

discursive relations ← DR^{-} ——— DR^{+} → Know-how

$$V(q) = q^{-2} + 2q^{-4} - 2q^{-5} + q^{-6} - 2q^{-7} + q^{-8}$$



Insights: Knots

discursive relations

DR -

Know-how without Mathematical representation and reasoning

Know-how with Mathematical representation and reasoning

DR+

Know-how

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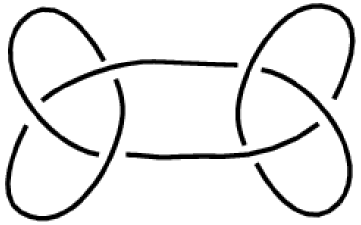


Insights: Knots

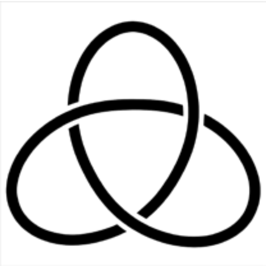
ontic relations

OR+

Know-what

left-handed trefoil knot

cannot be continuously deformed into



right-handed trefoil knot

OR-

Insights: Knots

ontic relations

OR+

OR-

Know-what

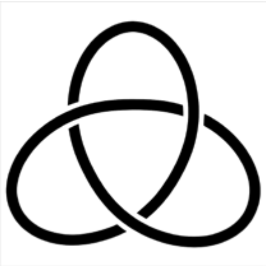
Jones polynomials for the left-handed and right-handed trefoils are, respectively

$V_l(q) = q^{-1} + q^{-3} - q^{-4}$ and $V_r(q) = q + q^3 - q^4$

These are not the same!

left-handed trefoil knot

cannot be continuously deformed into



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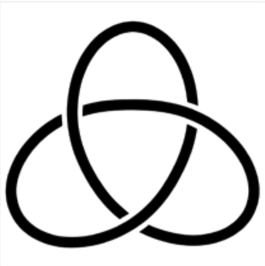

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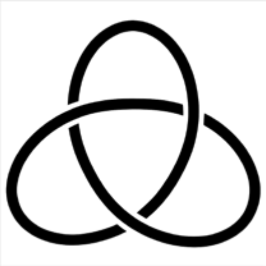


right-handed trefoil knot

ontic relations

OR+

Know-what

Know-why

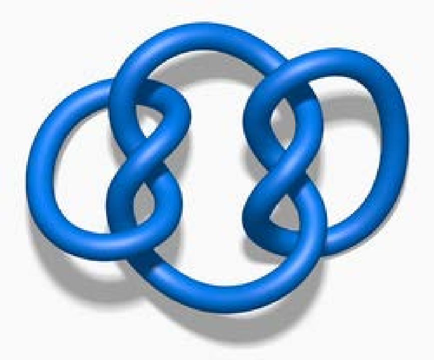
Know-that

OR-

Insights: Knots

ontic relations

OR+



Capturing the links

$$V(q) = (q^{-1} + q^{-3} - q^{-4})\,(q^{-1} + q^{-3} - q^{-4})$$

$$= q^{-2} + 2q^{-4} - 2q^{-5} + q^{-6} - 2q^{-7} + q^{-8}$$

situational

purist

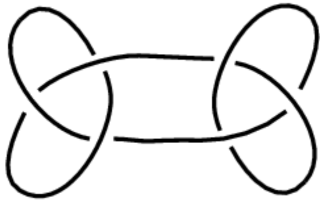
DR-

DR+

discursive relations

o/knower

doctrinal



Granny knot

OR-

Insights: Knots

Varying polynomial

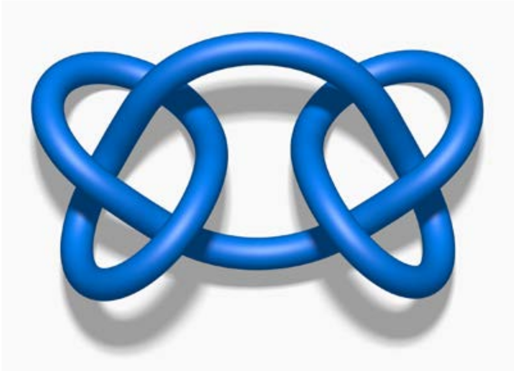
ontic relations

OR+

$(q + q^3 - q^4)$

$$V(q) = (q^{-1} + q^{-3} - q^{-4})\ \text{~~}(q^{-1} + q^{-3} - q^{-4})\text{~~}$$

$$= q^{-2} + 2q^{-4} - 2q^{-5} + q^{-6} - 2q^{-7} + q^{-8}$$



situational

purist

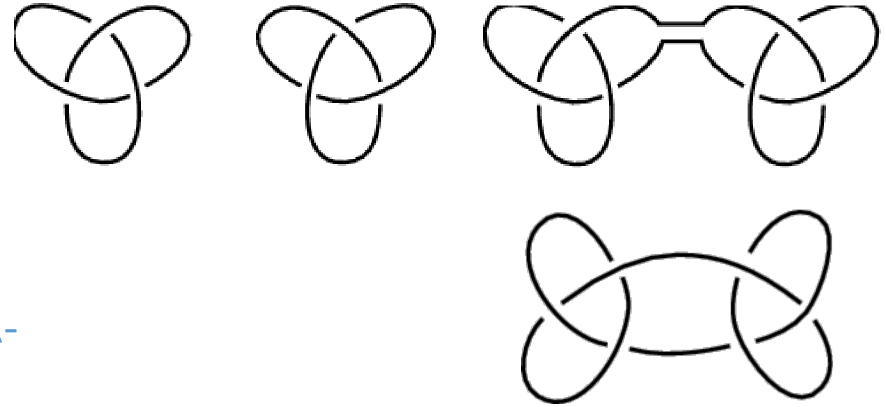
DR-

DR+

discursive relations

no/knower

doctrinal



OR-

A new knot

Research Methodology

Ontic Relations OR	*Know-what* relationship between a knowledge claim and the mathematical object of study	+ *stronger*	*Know-why* a mathematical claim holds for the mathematical object of study
		- *weaker*	*Know-that* a mathematical claim holds for the mathematical object of study
Discursive Relations DR	*Know-how* relationship between ways of referring to or reasoning about the mathematical object of study	+ *stronger*	*Know-how-with* Examples, representations, and reasoning from mathematics
		- *weaker*	*Know-how-without* Examples, representations, and reasoning without mathematics

Research Methodology

ontic relations

Know-what

OR+

Strategic competence refers to the ability to formulate (***non-routine***) mathematical problems, represent them, and solve them.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas and principles.

situational

purist

DR-

DR+

discursive relations

Know-how

no/knower

doctrinal

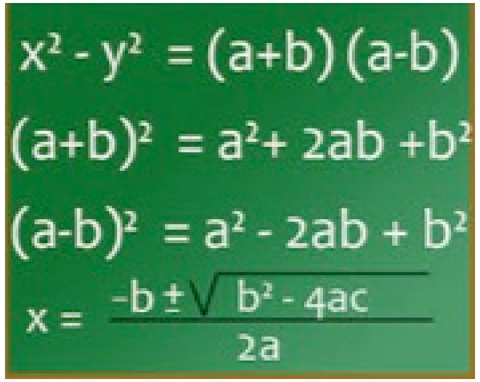
Intuitive understanding or individual interpretation and reasoning using no mathematical principles and techniques.

Procedure fluency in the sense of knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently for ***routine*** problems.

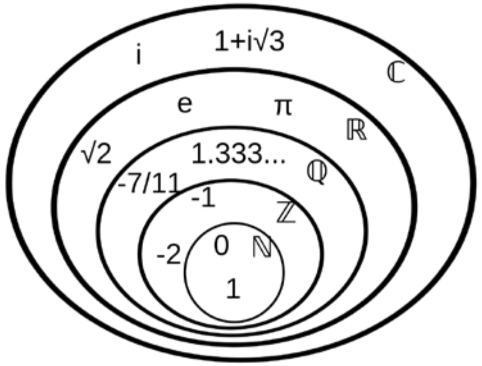
OR-

Insights of the Epistemic Plane of the Specialization dimension of Legitimation Code Theory (Maton 2014)

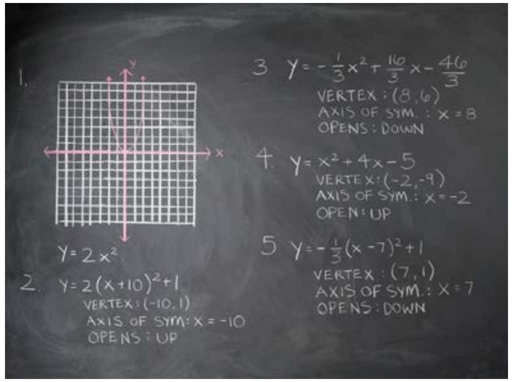
Five core areas of Mathematics



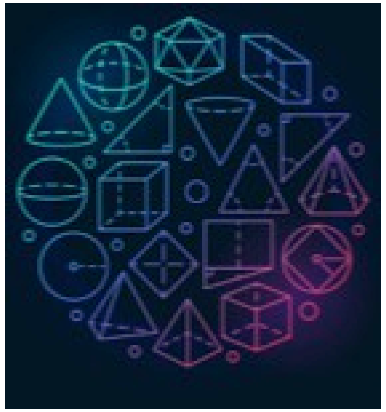
Algebraic Processes



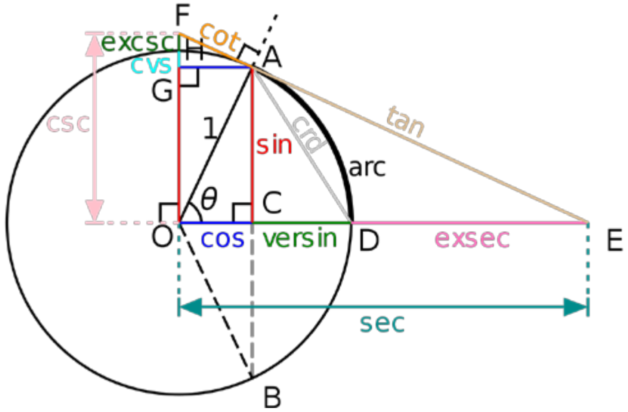
Number Sense



Functions



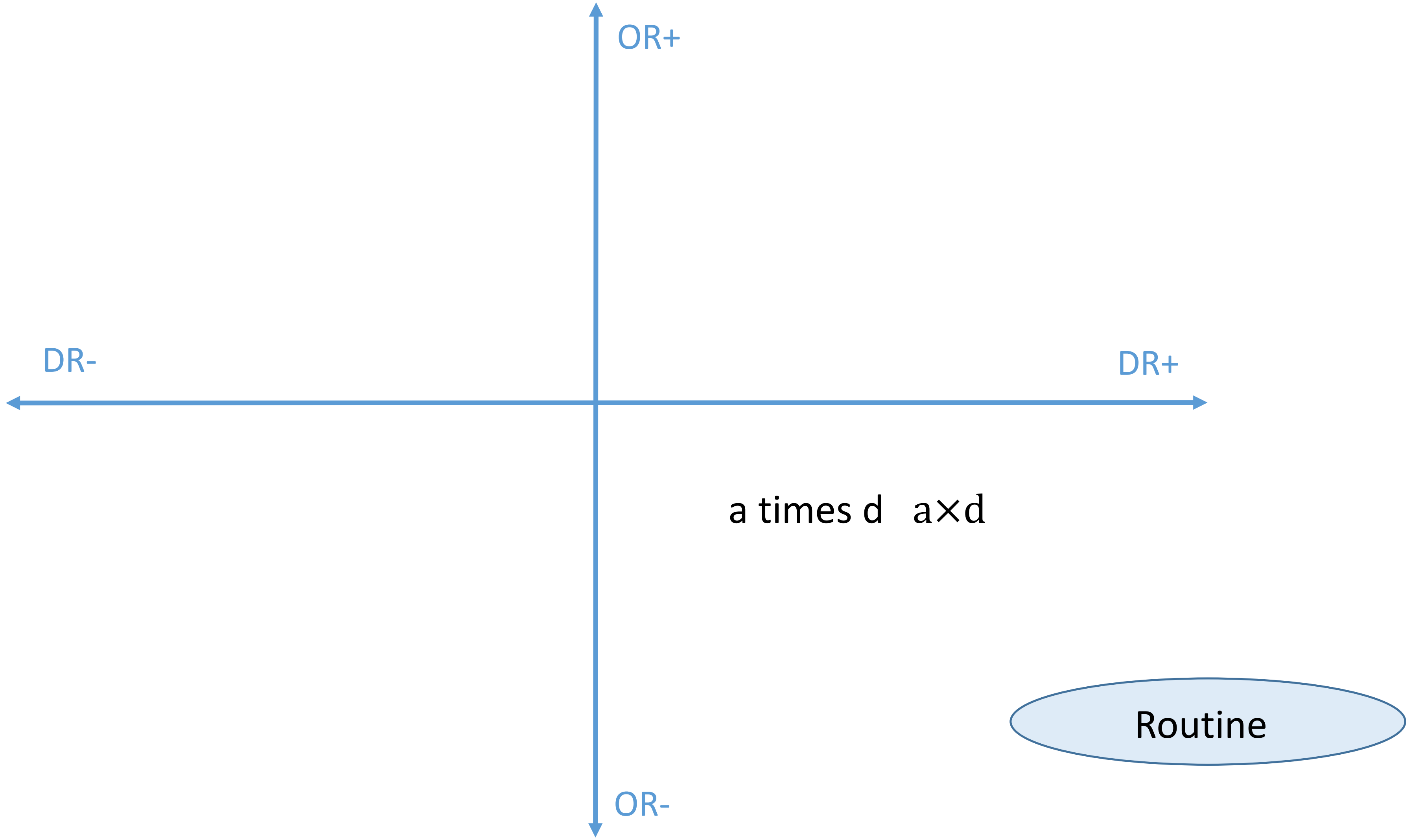
Geometry



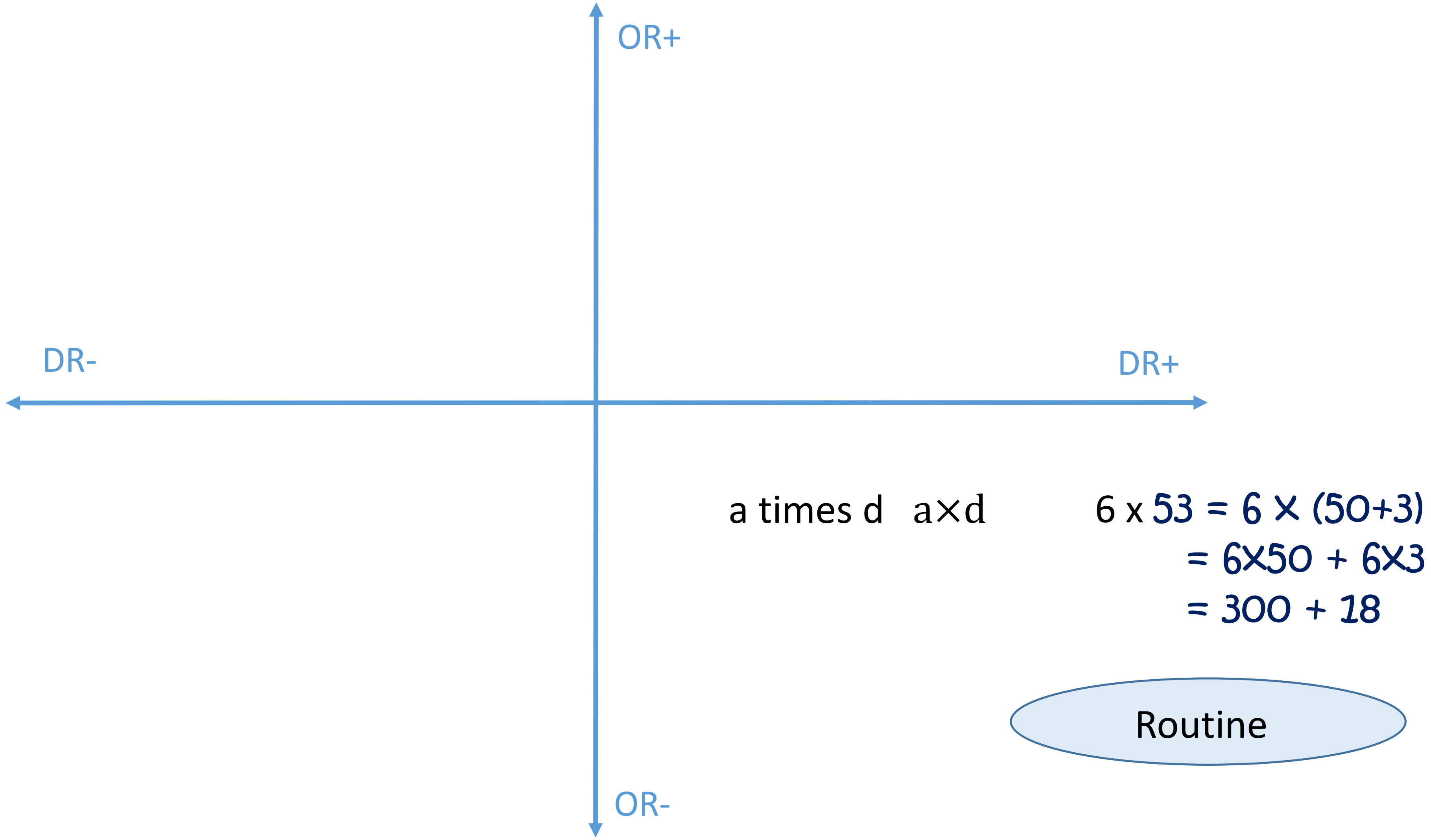
Trigonometry

NBT
National Benchmark Tests Project

Insights: Algebraic processes



Insights: Algebraic processes



Insights: Algebraic processes

OR+

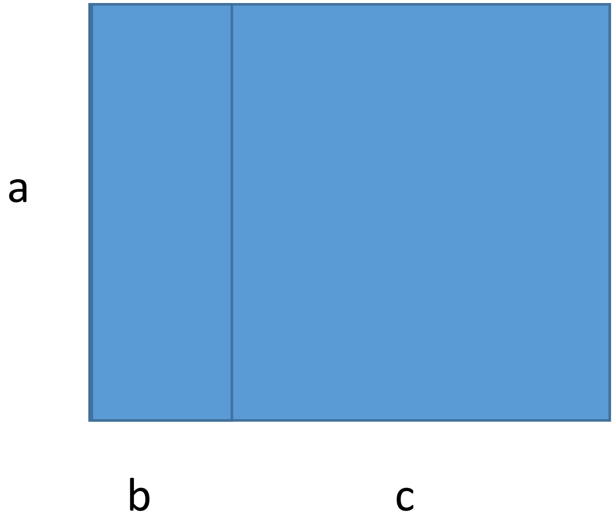
Distributivity Law

$a \times (b+c) = (a \times b) + (a \times c)$

$(b+c) \times a = (b \times a) + (c \times a)$

DR-

DR+



a times d $\quad a \times d$

$6 \times 53 = 6 \times (50+3)$

$= 6 \times 50 + 6 \times 3$

$= 300 + 18$

Routine

OR-

Insights: Algebraic processes

OR+

Non-routine

Set theory: intersection (∩), union (∪)
Logic: conjunction (∧), disjunction(∨)

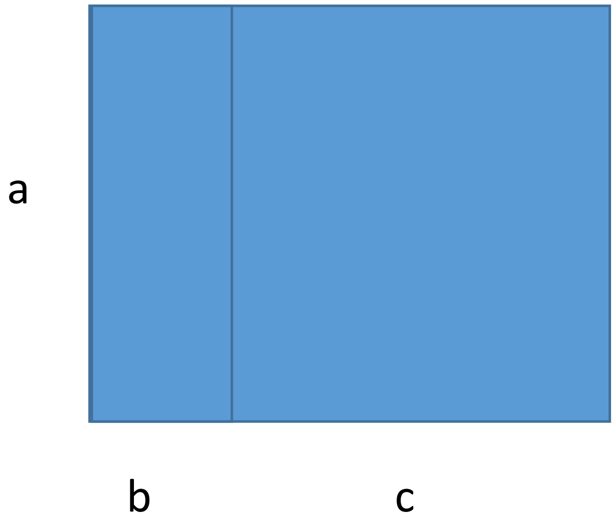
Distributivity Law

$a\times(b+c) = (a\times b) + (a\times c)$

$(b+c)\times a = (b\times a) + (c\times a)$

DR-

DR+



a times d $a\times d$

6 x 53 = 6 × (50+3)
= 6×50 + 6×3
= 300 + 18

Routine

OR-

Insights: Number sense

Equivalence relation
Partial relation

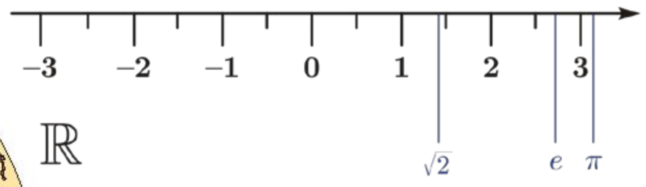
Logical equivalence
Isomorphism

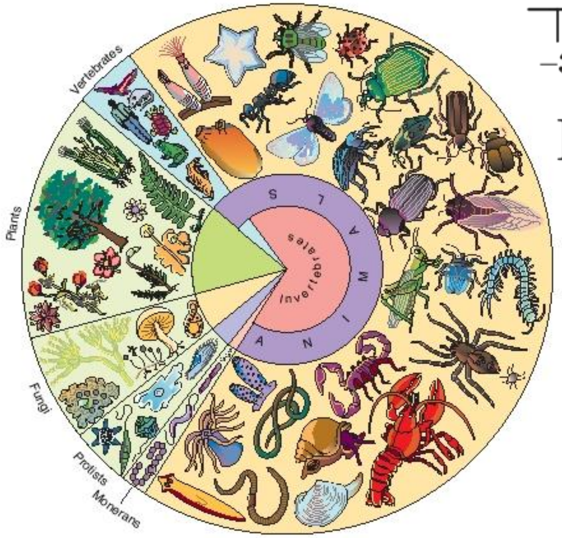
OR+

For x,y,z in X
- x=x (reflexivity)
- x=y implies y=x (symmetry)
- x=y and y=z implies x=z (transitivity)

DR-

DR+



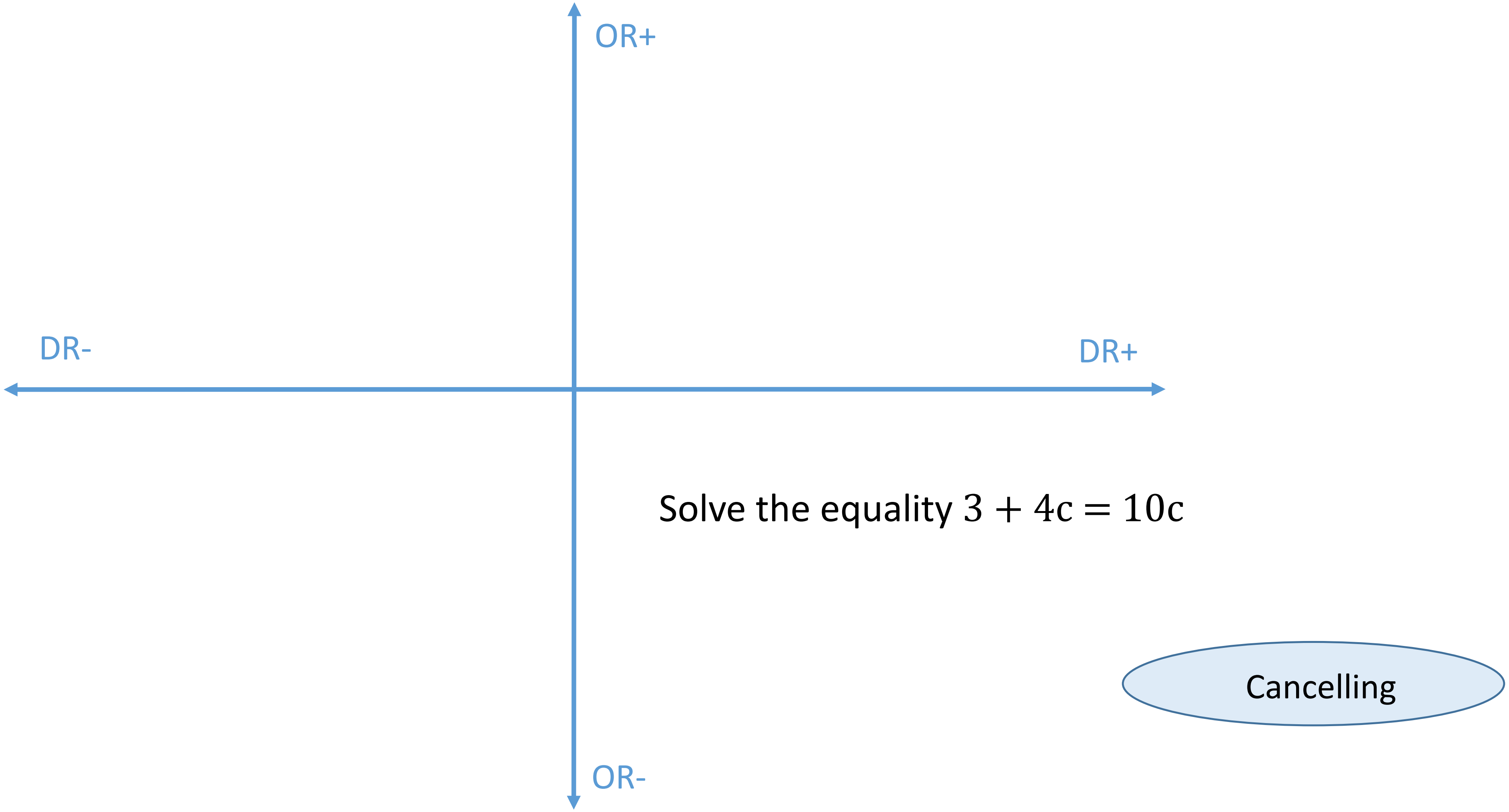


x equal to y $x = y$

x less than y $x < y$

OR-

Example : Algebraic processes & number sense



Example : Algebraic processes & number sense

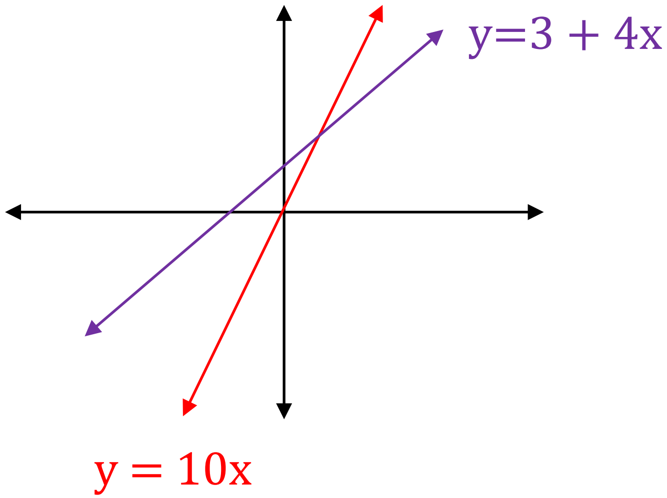
OR+

$a=b$ iff $a \times c = b \times c$
$a=b$ iff $a + c = b + c$

If $c>0$ then $a<b$ iff $a \times c < b \times c$
If $c<0$ then $a<b$ iff $a \times c > b \times c$

DR-

DR+

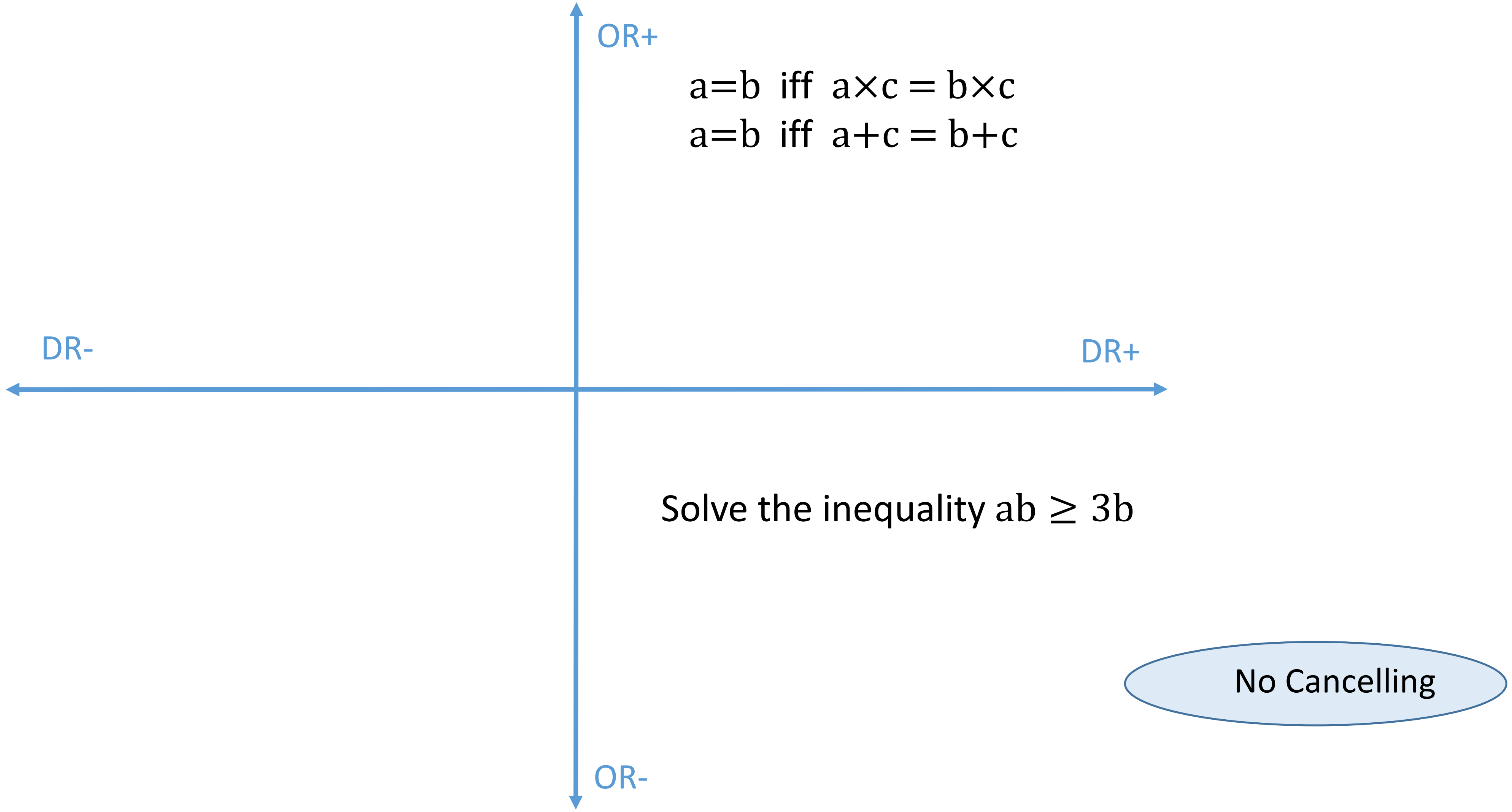


Solve the equality $3 + 4c = 10c$

Cancelling

OR-

Example : Algebraic processes & number sense



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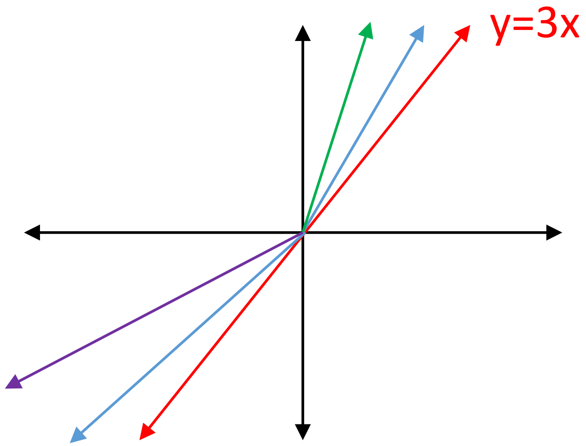
DR-

DR+



Solve the inequality $ab \geq 3b$

No Cancelling

OR-

Example : Algebraic processes & number sense

OR+

For any real number x, which one of the following statements is always true?

$-x < 0$

$\frac{1}{x}$ is rational

$\frac{x}{x+1} < 1$

$\frac{1}{x} > 1$ if $0 < x < 1$

a=b iff a×c = b×c

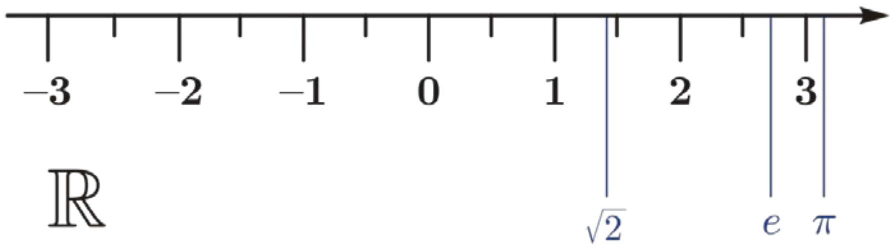
a=b iff a+c = b+ c

If c>0 then a<b iff a× c < b×c

If c<0 then a<b iff a×c > b×c

DR-

DR+



Solve the inequality $ab \geq 3b$

No Cancelling

OR-

Insights: Functions

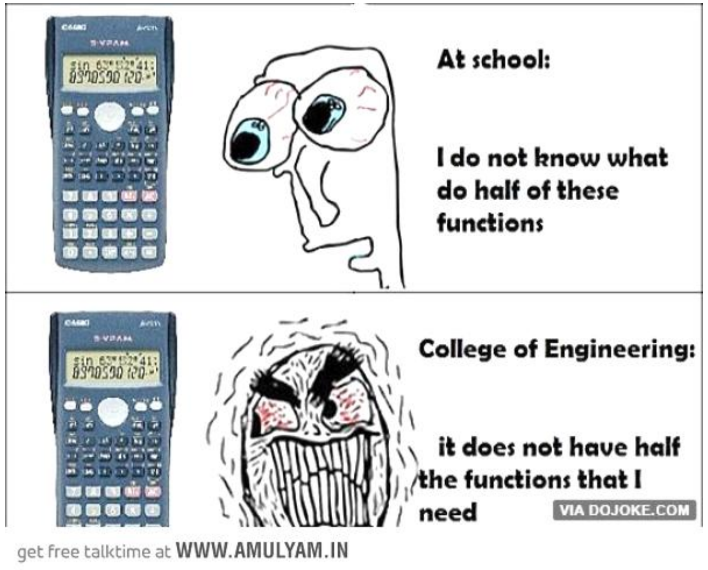
OR+

Multivariate functions
Multi-valued functions
Functional programming

A function $f \subseteq X \times Y$ from X to Y is such that for each $x \epsilon X$ there is at most one $y \epsilon Y$ with $(x, y) \epsilon f$.

DR-

DR+



A function is *rule* that *assigns* to each element of one set (its domain) exactly one element in another set.

OR-

Insights: Functions

OR+

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A function $f \subseteq X \times Y$ from X to Y is such that
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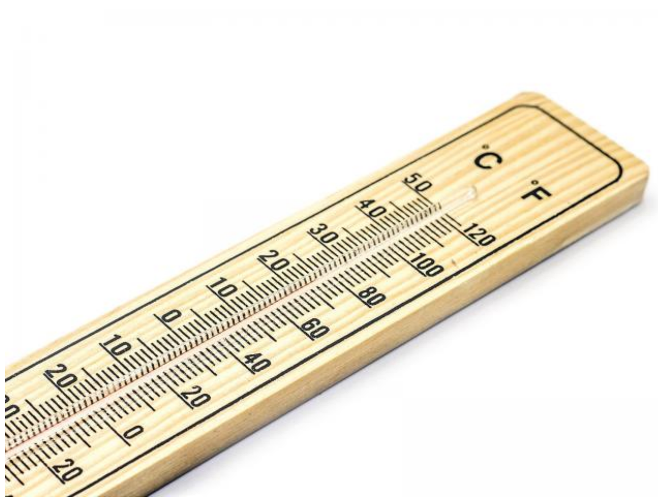
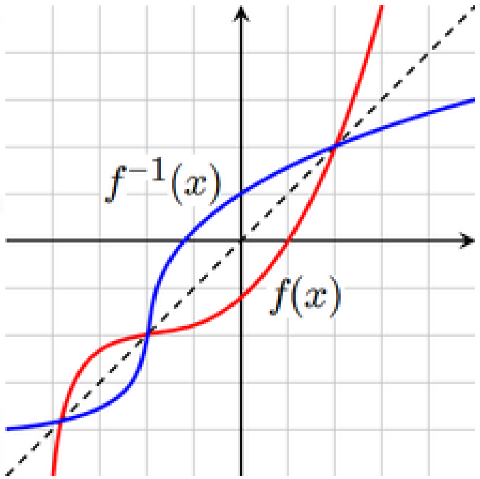
For any injective function $f \subseteq X \times Y$, its inverse function $f^{-1} \subseteq Y \times X$ is defined by

$$x = f^{-1}(y) \Leftrightarrow f(x) = y$$

DR-

DR+





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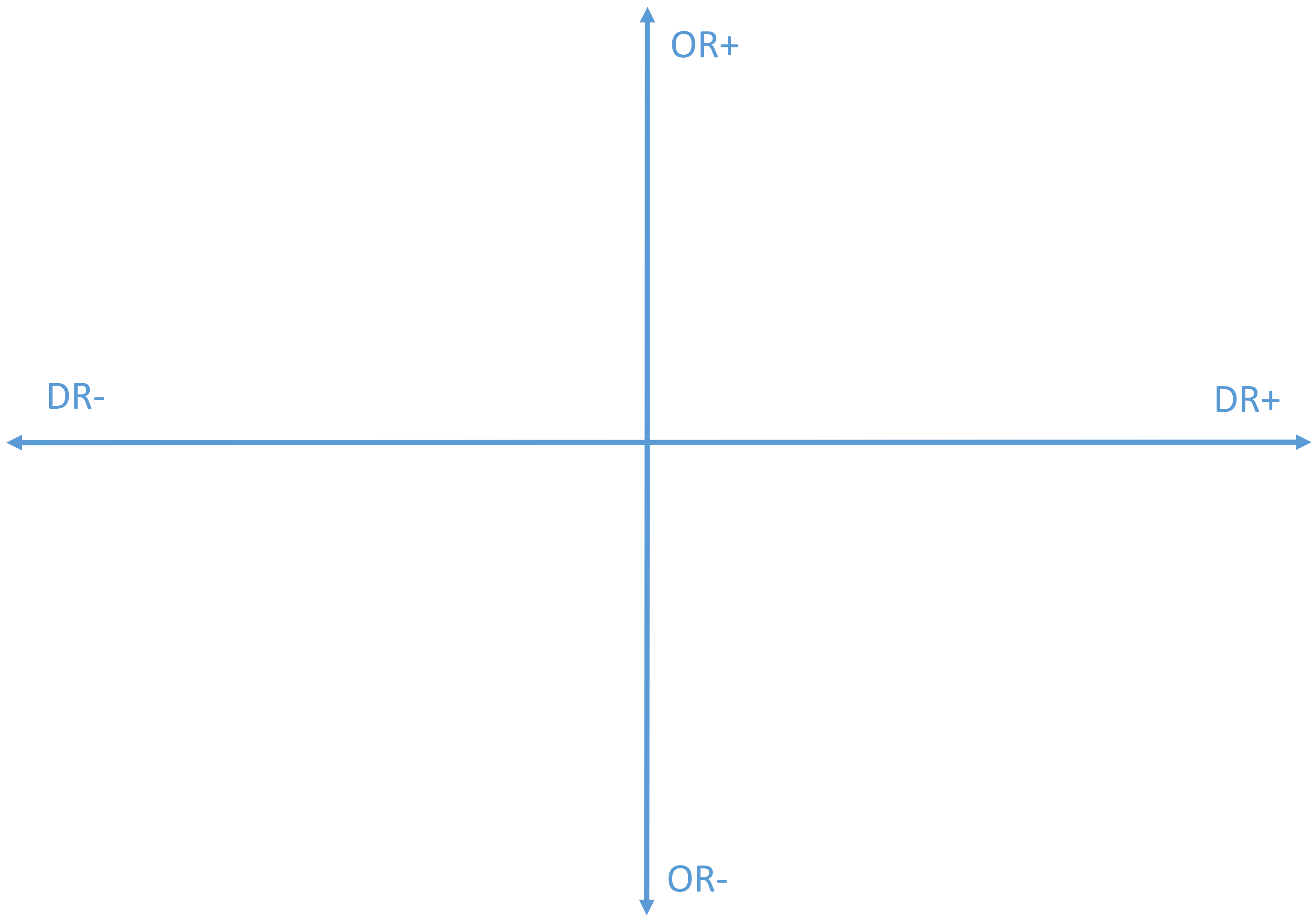
Procedure for finding inverse function of function f :
Let $y = f(x)$. Solve for x. The resulting function $g(y)$ is the inverse of function $f(x)$.

OR-

Example : Functions

A cycle shop has 36 bicycles and tricycles in stock.
Collectively there are 80 wheels.
How many bicycles and how many tricycles are there?



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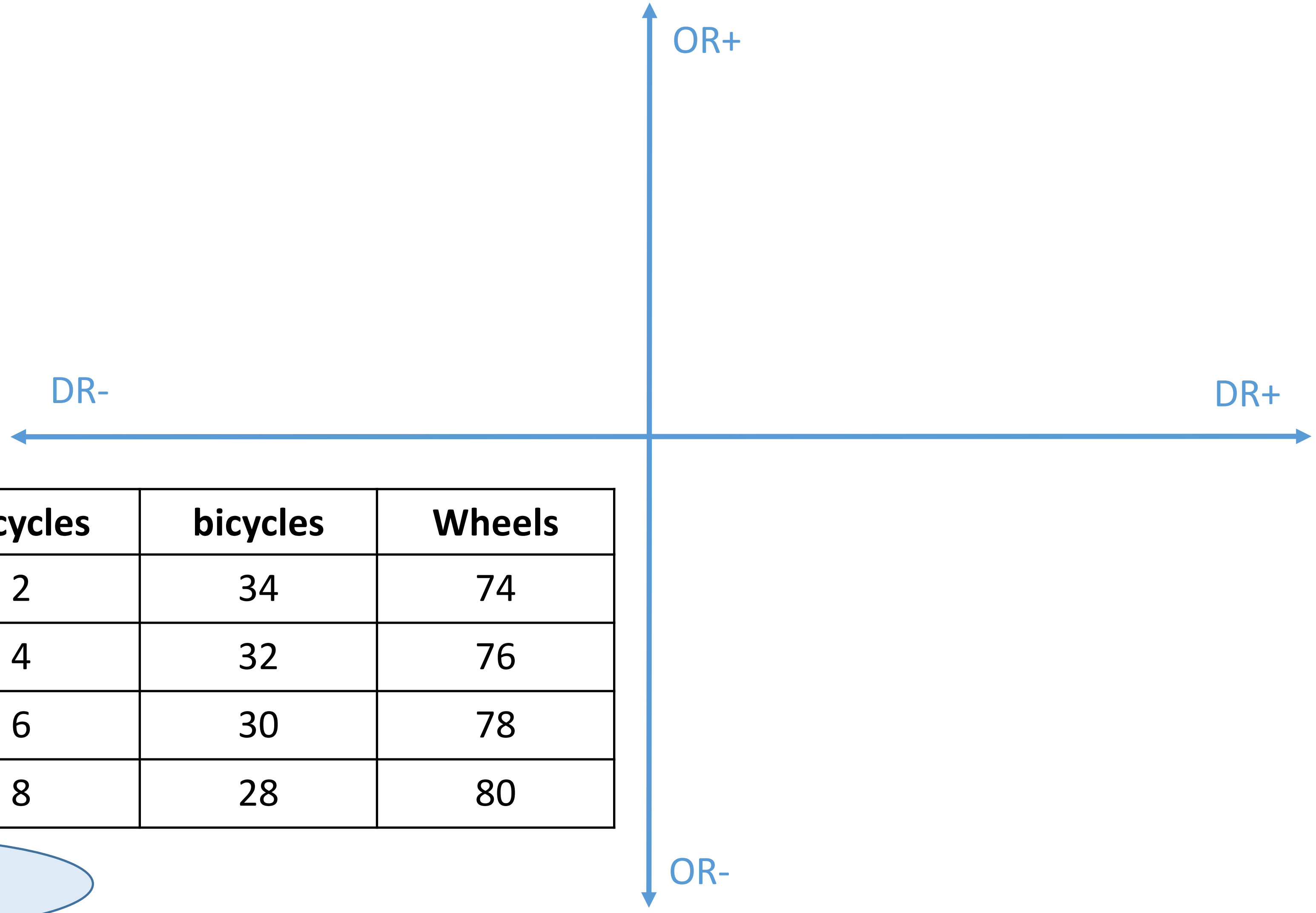


tricycles	bicycles	Wheels
2	34	74
4	32	76
6	30	78
8	28	80

Trial and error

Example : Functions

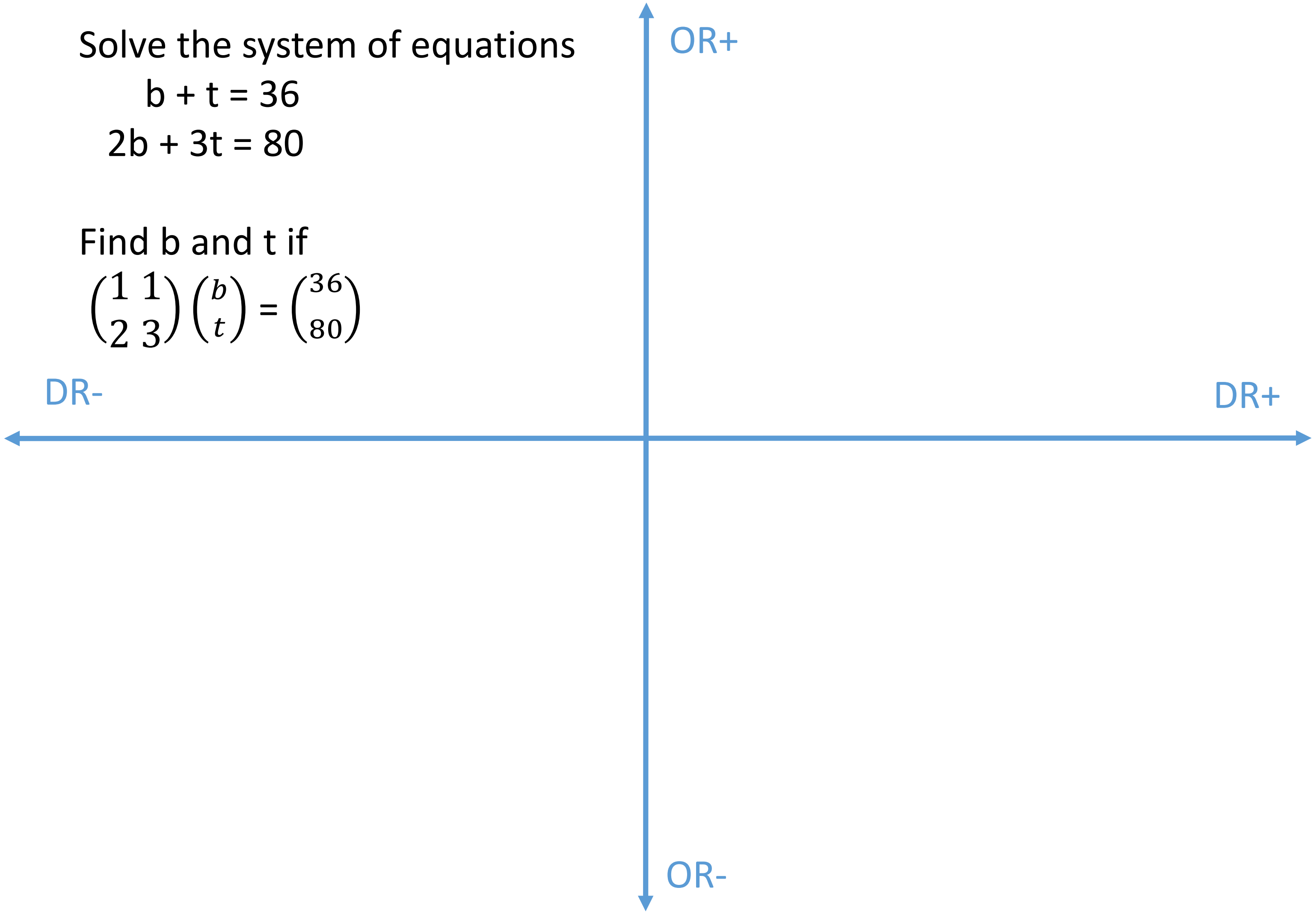
A cycle shop has 36 bicycles and tricycles in stock.
Collectively there are 80 wheels.
How many bicycles and how many tricycles are there?

Solve the system of equations

$$b + t = 36$$
$$2b + 3t = 80$$

Find b and t if

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} b \\ t \end{pmatrix} = \begin{pmatrix} 36 \\ 80 \end{pmatrix}$$



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OR+

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DR-

DR+

B9 f_x {=MINVERSE(B6:C7)}

	A	B	C	D	E	F	G
1	**System of Equations**						
2							
3		b + t = 36					
4		2b+3t = 80					
5							
6		1	1		b		36
7	A =	2	3	x =	t	y =	80
8							
9		3	-1				
10	Inverse of A =	-2	1				
11							
12							

$$\left[\begin{array}{cc|cc}1 & 1 & 1 & 0\\ 2 & 3 & 0 & 1\end{array}\right]$$

Gauss elimination

$$\left[\begin{array}{cc|cc}1 & 0 & 3 & -1\\ 0 & 1 & -2 & 1\end{array}\right]$$

OR-

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OR+

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For any n x n invertible matrix A

$$\underline{x} = A^{-1}\underline{y} \Leftrightarrow A\underline{x} = \underline{y}$$

Inverse of 2x2 matrix A is

$$A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

DR-

DR+

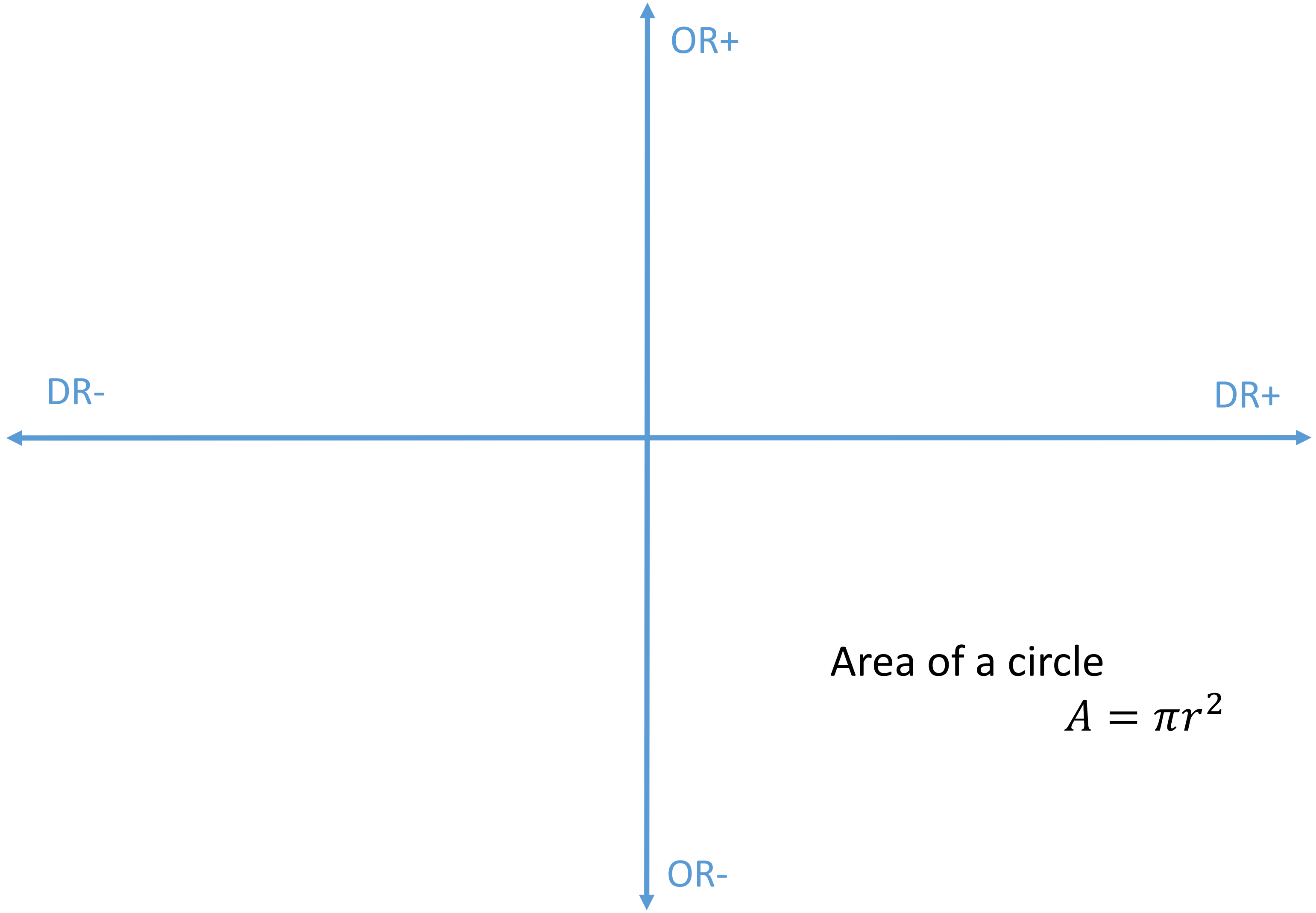
B9 {=MINVERSE(B6:C7)}

	A	B	C	D	E	F	G
1	**System of Equations**						
2							
3		b + t = 36					
4		2b+3t = 80					
5							
6		1	1		b		36
7	A =	2	3	x =	t	y =	80
8							
9		3	-1				
10	Inverse of A =	-2	1				
11							
12							

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{array}\right] \xrightarrow{\text{Gauss elimination}} \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

OR-

Insights : Geometric Concepts

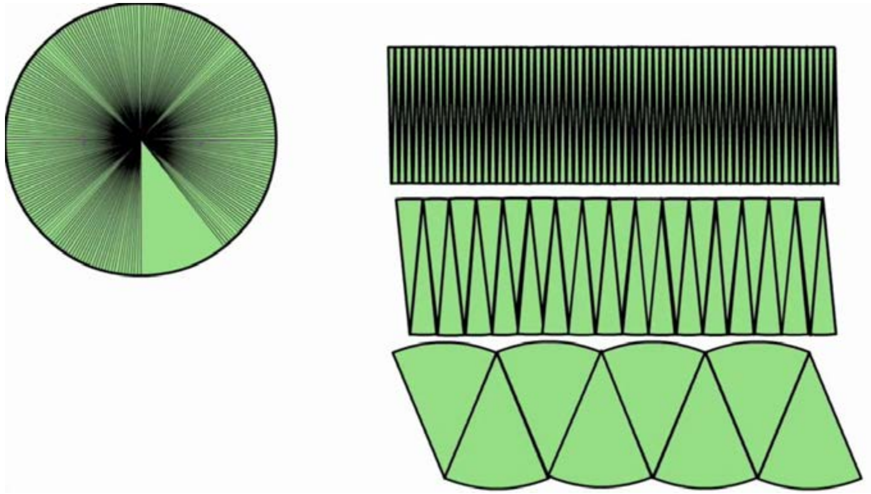


Insights : Geometric Concepts

OR+

DR-

DR+



Area of a circle

$$A = \pi r^2$$

OR-

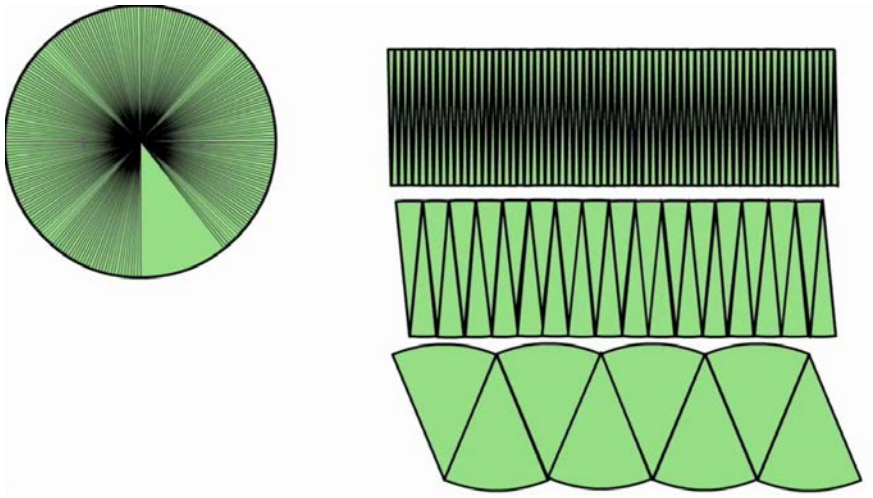
Insights : Geometric Concepts

OR+

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(x_k)\Delta x$$

DR-

DR+



Area of a circle

$$A = \pi r^2$$

OR-

Insights : Geometric Concepts

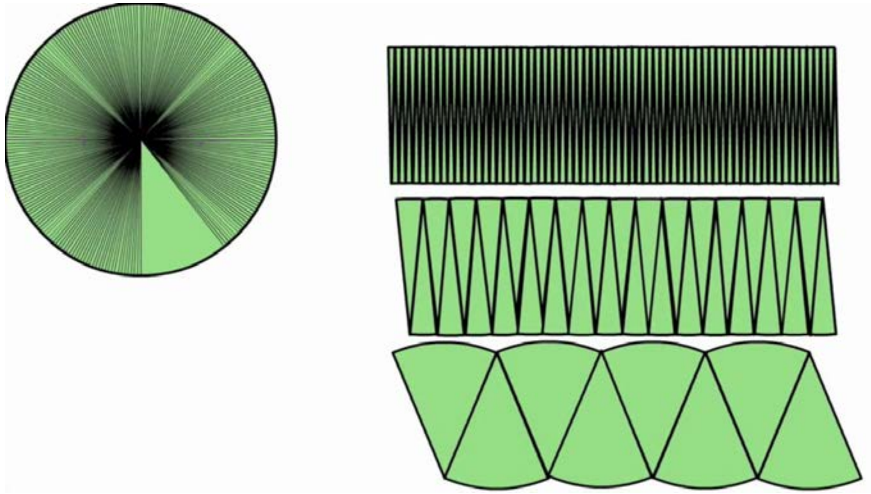
OR+

Area of other shapes
Volume of other shapes
Area under a curve
Volume of a surface

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=0}^{n-1} f(x_k)\Delta x$$

DR-

DR+



Area of a circle

$$A = \pi r^2$$

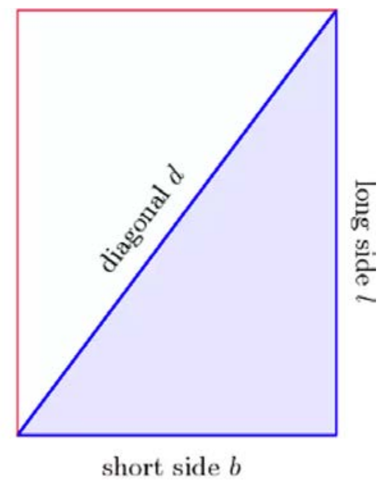
OR-

Insights : Trigonometry

Reciprocal and Quotient Identities	$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\tan\theta = \frac{1}{\cot\theta} = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$
Pythagorean Identities	$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$
Sum and Difference Identities	$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$
Double Angle Identities	$\sin(2A) = 2\sin A\cos A$ $\cos(2A) = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$ $\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$
Half Angle Identities	$\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$ $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$ $\tan\left(\frac{A}{2}\right) = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Insights : Trigonometry = triangle + measure

OR+



Pythagorean triples

$(b/l)^2 + 1 = (d/l)^2$

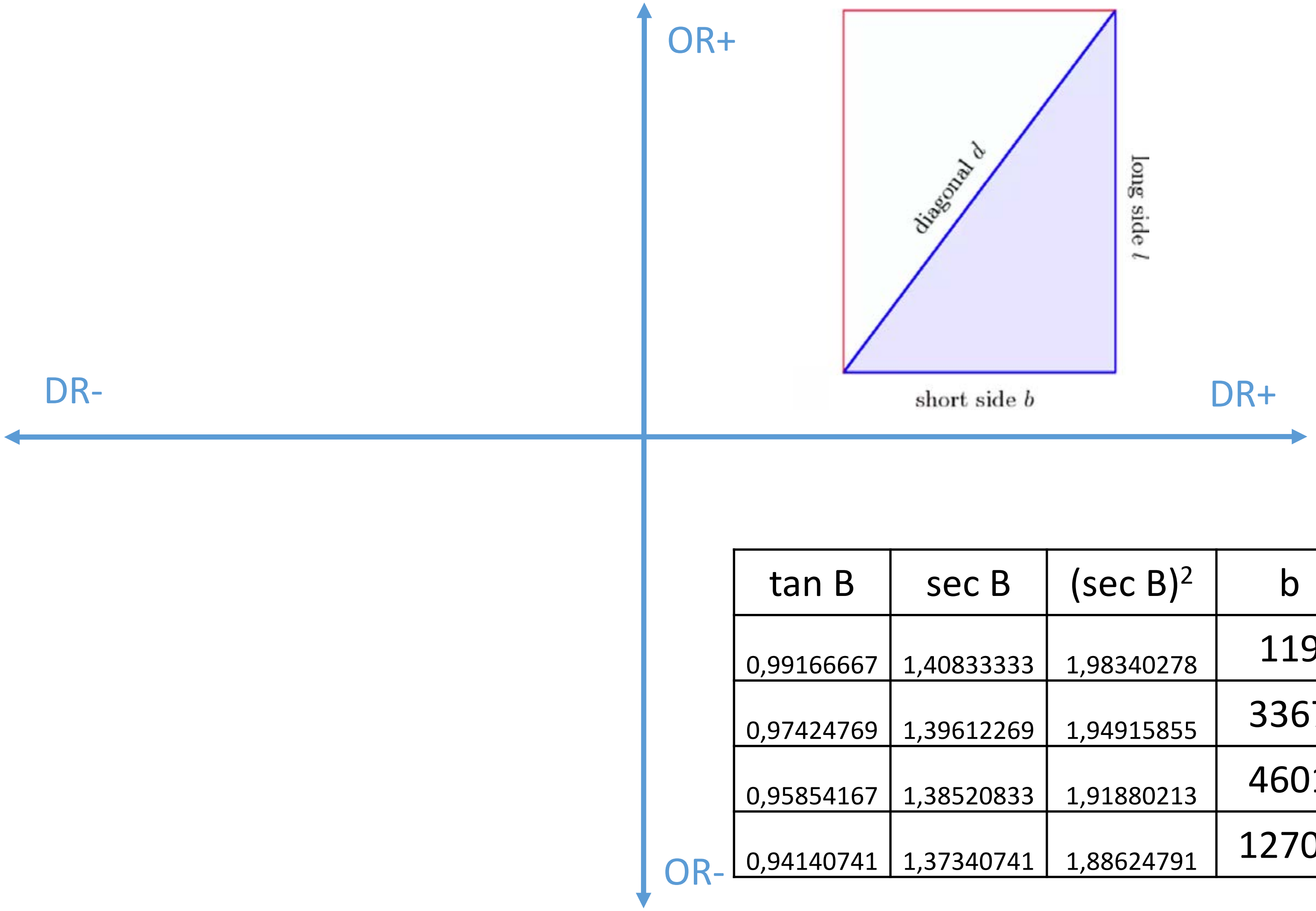
DR-

DR+

b/l	d/l	$(d/l)^2$	b	d	l
0,99166667	1,40833333	1,98340278	119	169	120
0,97424769	1,39612269	1,94915855	3367	4825	3456
0,95854167	1,38520833	1,91880213	4601	6649	4800
0,94140741	1,37340741	1,88624791	12709	18541	13500

OR-

Insights : Trigonometric Functions - angles ito sides



Pythagorean identities

$\tan^2 B + 1 = \sec^2 B$

tan B = b/l

sec B = d/l

tan B	sec B	(sec B)^2	b	d	l
0,99166667	1,40833333	1,98340278	119	169	120
0,97424769	1,39612269	1,94915855	3367	4825	3456
0,95854167	1,38520833	1,91880213	4601	6649	4800
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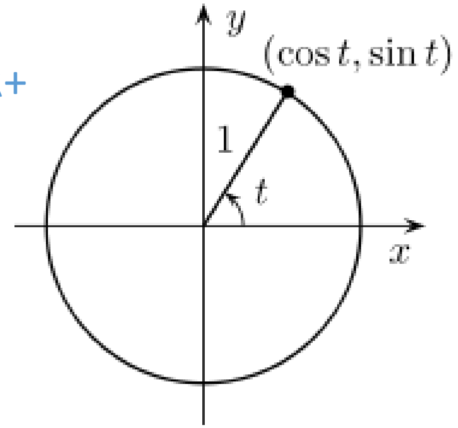
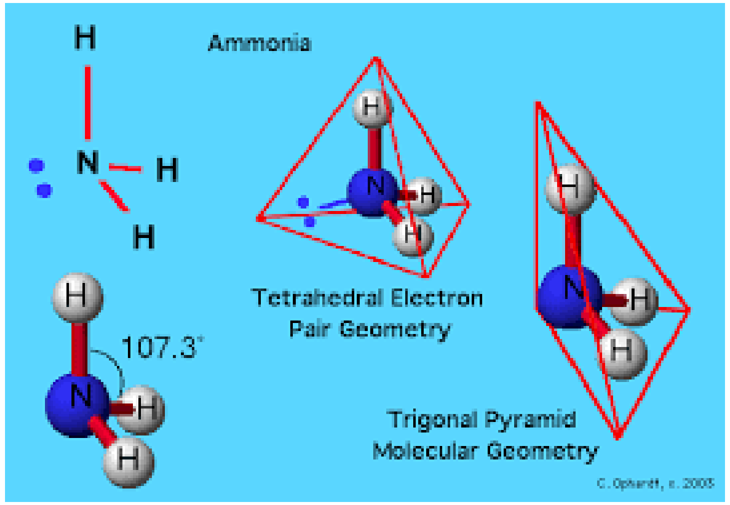
Insights : Trigonometric Functions - angles ito sides

OR+



DR-

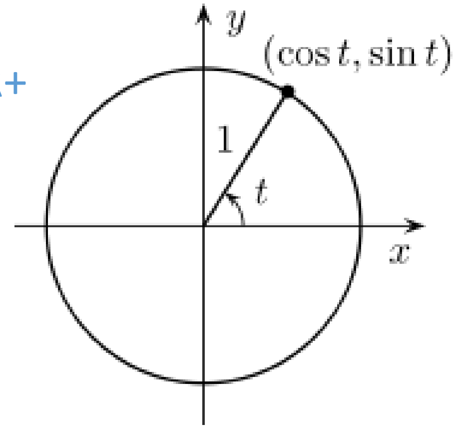
DR+

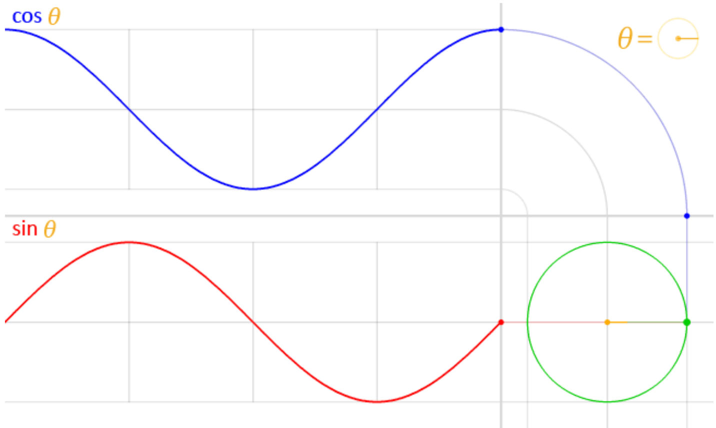


OR-

Insights : Trigonometric Functions - angles ito sides

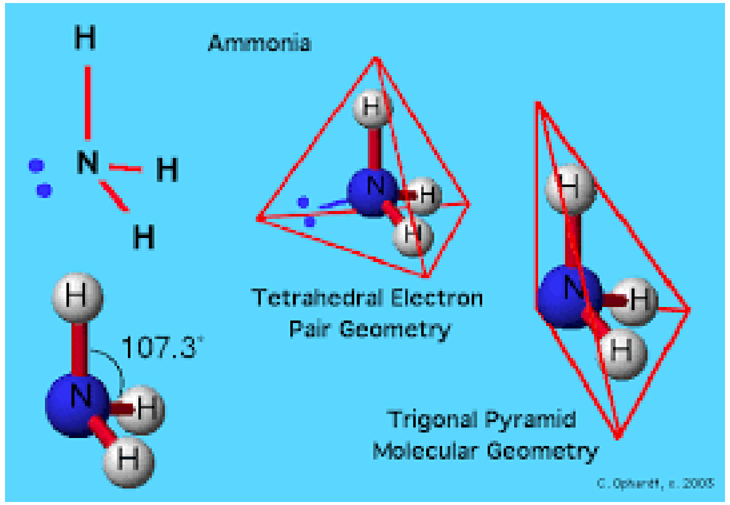
OR+





DR-

DR+

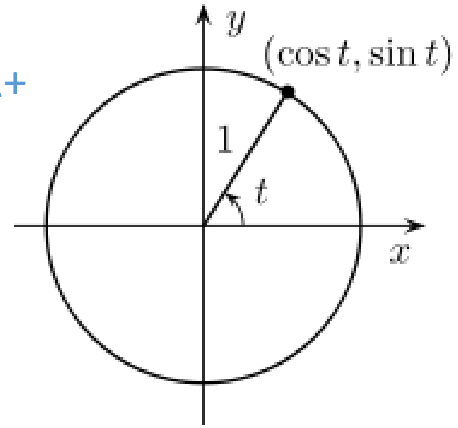


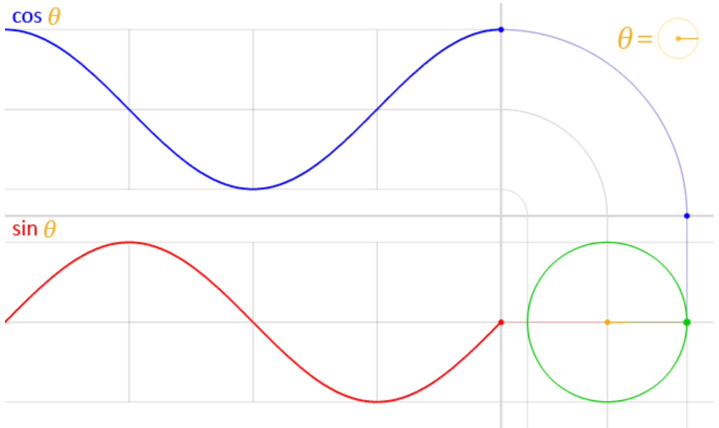
Sum and Difference Identities	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$ $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$	
Double Angle Identities	$\sin(2A)=2\sin A\cos A$	$\cos(2A)=\cos^2 A-\sin^2 A$ $=1-2\sin^2 A$ $=2\cos^2 A-1$	$\tan(2A)=\dfrac{2\tan A}{1-\tan^2 A}$
Half Angle Identities	$\sin\left(\dfrac{A}{2}\right)=\pm\sqrt{\dfrac{1-\cos A}{2}}$	$\cos\left(\dfrac{A}{2}\right)=\pm\sqrt{\dfrac{1+\cos A}{2}}$	$\tan\left(\dfrac{A}{2}\right)=\dfrac{\sin A}{1+\cos A}=\dfrac{1-\cos A}{\sin A}$

OR-

Insights : Trigonometric Functions - angles ito sides

OR+

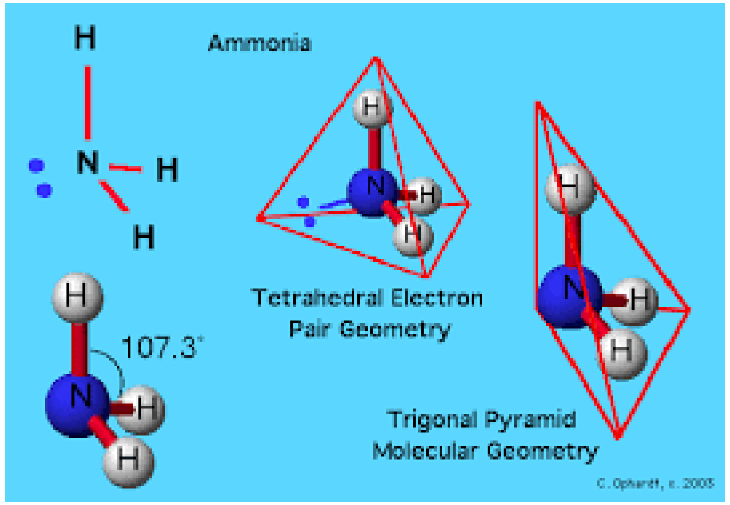




Radian measure
Trigonometric functions in Cartesian plane
Trigonometric functions as power series
Complex numbers

DR-

DR+



OR-

Sum and Difference Identities	$\sin(A+B)=\sin A\cos B+\cos A\sin B$ $\sin(A-B)=\sin A\cos B-\cos A\sin B$ $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $\cos(A-B)=\cos A\cos B+\sin A\sin B$ $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$	
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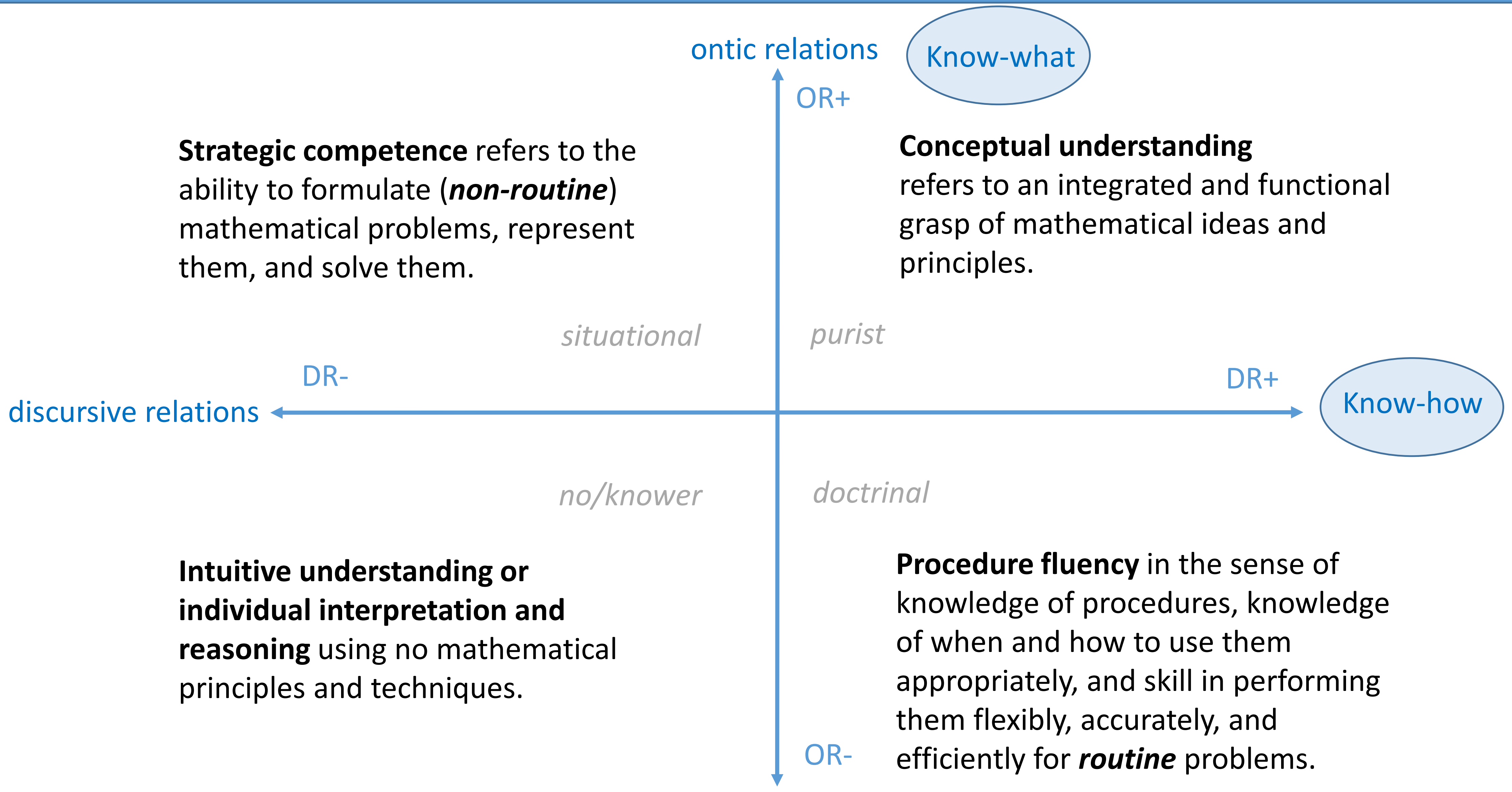
Insights of the Epistemic Plane of the Specialization dimension of Legitimation Code Theory (Maton 2014)

Orthogonality is key



Insights of the Epistemic Plane of the Specialization dimension of Legitimation Code Theory (Maton 2014)

Insights: NBT basic achievement level

ontic relations

OR+

Strategic competence refers to the ability to formulate (***non-routine***) mathematical problems, represent them, and solve them.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas and principles.

situational

purist

DR-

DR+

discursive relations

no/knower

doctrinal

Intuitive understanding or individual interpretation and reasoning using no mathematical principles and techniques.

Procedure fluency in the sense of knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently for ***routine*** problems.

OR-

Insights: NBT intermediate achievement level

ontic relations

OR+

Strategic competence refers to the ability to formulate (***non-routine***) mathematical problems, represent them, and solve them.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas and principles.

situational

purist

DR-

DR+

discursive relations

no/knower

doctrinal

Intuitive understanding or individual interpretation and reasoning using no mathematical principles and techniques.

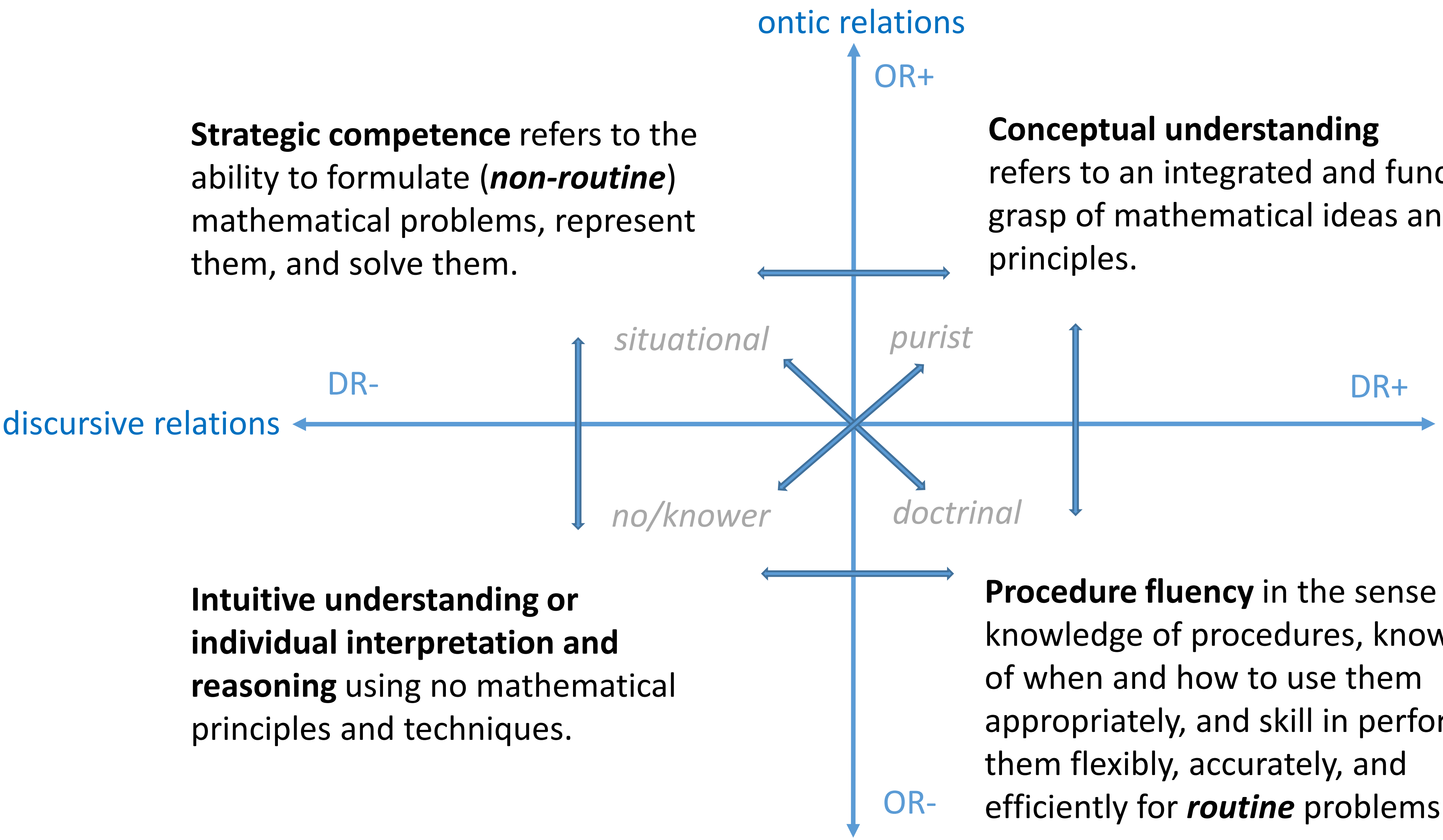
Procedure fluency in the sense of knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently for ***routine*** problems.

OR-

Insights: NBT proficient achievement level

Strategic competence refers to the ability to formulate (***non-routine***) mathematical problems, represent them, and solve them.

Conceptual understanding refers to an integrated and functional grasp of mathematical ideas and principles.



Intuitive understanding or individual interpretation and reasoning using no mathematical principles and techniques.

Procedure fluency in the sense of knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently for ***routine*** problems.

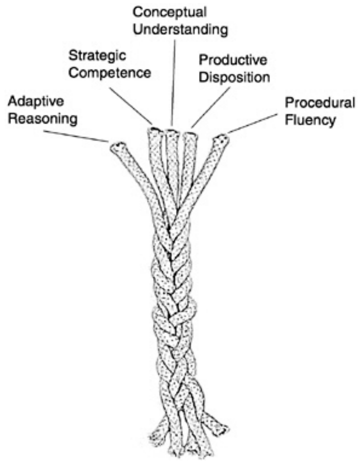
Insights of the Epistemic Plane of the Specialization dimension of Legitimation Code Theory (Maton 2014)

Navigating between the insights

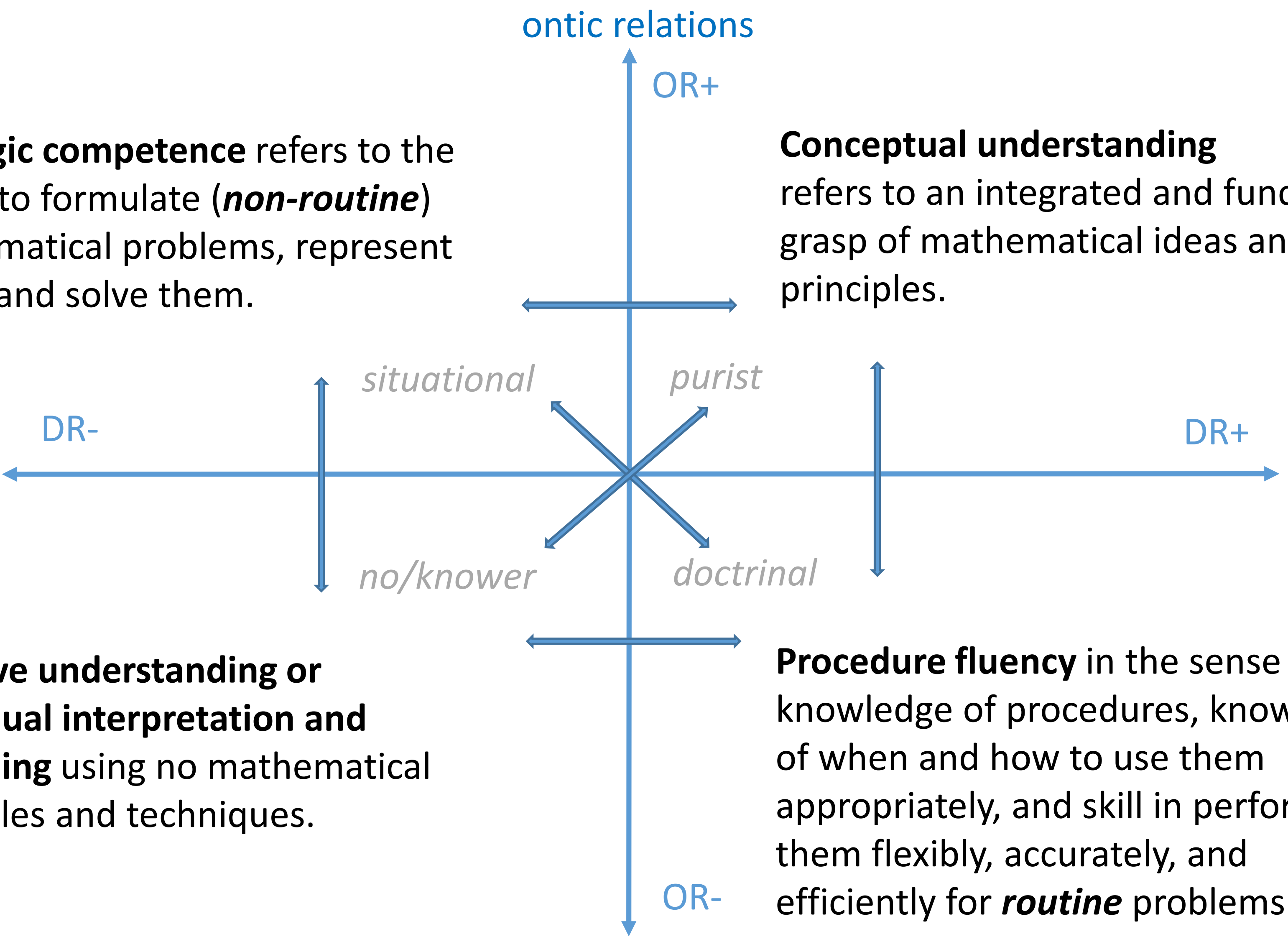
This has been the inspiration for offering to first-year Mathematics students

curriculum integrated differentiated support,

which is developing further to incorporate individualised support using learning technologies …



Navigating between the insights



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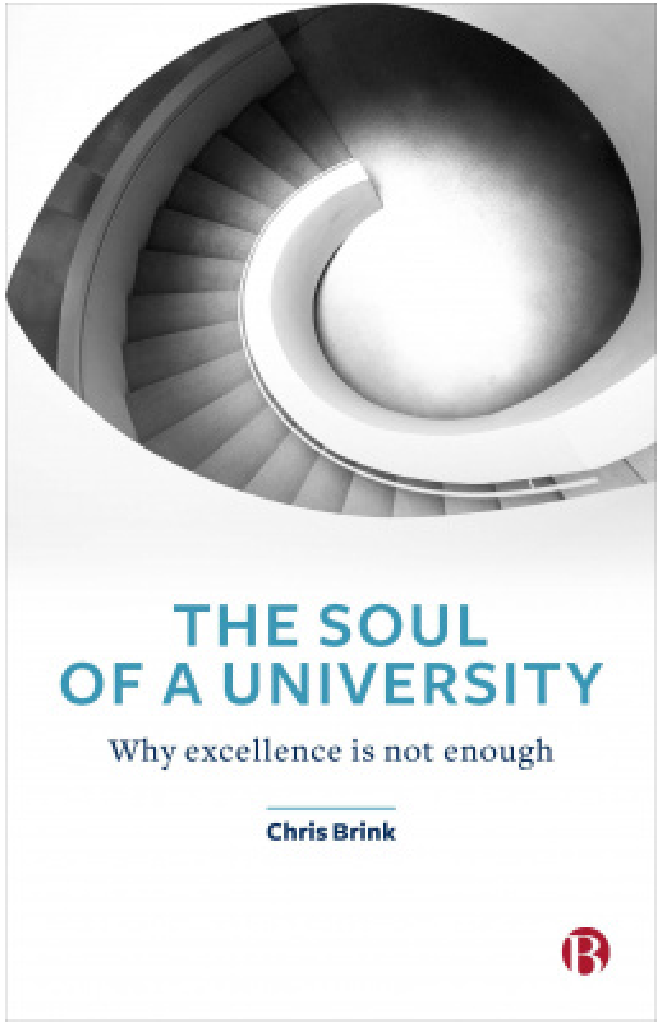
Insights from Legitimation Code Theory (Maton 2014) by Rewitzky (2019)

S100 1918·2018

SCIENCE
NATUURWETENSKAPPE
EYOBUNZULULWAZI



We have argued that different forces [‘what’ vs ’how’] can, however, become quite productive when we think two-dimensionally rather than linearly, and dynamically rather than statically.

Two-dimensional thinking,
in turn, requires
orthogonal axes of thought.

So, orthogonality is the key idea.

The Soul of a University: Why excellence is not enough (Chris Brink, 2018).